The Pernicious Influence of Mathematics on Science

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Our announced subject today is the role of mathematics in the formulation of physical theories. I wish, however, to take a slight detour from the main topic, that is, the interaction of science and mathematics, which I wish to discuss in two regards: in the first place, to confine, my self to the negative aspects of this interaction, leaving to others the treatment of the amazing triumphs of the mathematical method; in the second place, to concentrate not only on the negative aspects but also on the positive aspects of the mathematical method in the context of the new physics, which I wish to discuss in the negative aspect of this interaction. The latter, which may appear to make a certain remark about computer science, is not in the province of pure mathematics. However, I wish to reserve an appropriate discussion of computer science for another occasion.

I wish, then, to confine myself to the negative aspects of the interaction of science and mathematics.

In the area of computer science, a student of integral calculus may be forced to use the notion of computer science in the formulation of the mathematical notion of a computer, in a computer science context. Such use may be very misleading, as the student of integral calculus is not familiar with the notion of computer science, which I wish to discuss in a separate context.

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SCHWARZ: The Pernicious Influence of Mathematics

The influence of mathematics depends on an initial mathematical stage of the problem. This stage is critical because the assumptions are then used too literally. Mathematics is often used in science, and even in engineering, to provide a framework for theoretical assumptions that may be incorrect. Mathematics offers a precise and stable solution to problems, but only if the assumptions are correct. A common example is the Dirac delta function, which was originally thought to be a function, but it later became clear that it was not a function in the conventional sense. This illustrates the danger of using mathematics too literally, as it can lead to incorrect conclusions.

The use of mathematics in science is problematic because it often provides a framework that is too rigid. Mathematics is used to provide a precise solution, but this solution can be misleading if the assumptions are incorrect. For example, the Dirac delta function was used in physics to represent an impulse, but it later became clear that it was not a function in the traditional sense. This highlights the need for caution when using mathematics in science.

In conclusion, the influence of mathematics on science is complex, and it is important to be aware of its limitations. Mathematics should be used as a tool to provide a framework for scientific inquiry, but it should not be taken too literally, as this can lead to incorrect conclusions.
This is a continuation of the previous text.

The physical sciences, upon a few dominant simple factors. Beyond the well-traveled path, mathematics loses its inapplicable combinatorial particularities. Thus, the mathematical technique can only reach far if it starts from a point close to the simple essentials of a problem which has simple essentials. That form of wisdom which is the opposite of single-mindedness, the ability to keep many threads in hand, to draw for an argument from any disparate sources, is quite foreign to mathematics. This inability accounts for much of the difficulty which mathematics experiences in attempting to penetrate the social sciences. We may perhaps attempt a mathematical economics — how difficult would be a mathematical history! Mathematics adjusts only with reluctance to the external, and vitally necessary, approximating of the scientist, and shudders each time a batch of small terms is cavalierly erased. Only with difficulty does it find its way to the scientist's ready grasp of the relative importance of many factors. Quite typically, science leaps ahead and mathematics plods behind.

Related to this deficiency of mathematics, and perhaps more productive of rueful consequence, is the single-mindedness of mathematics — its willingness, like that of a computing machine, to elaborate upon any idea, however absurd; to dress scientific brilliancies and scientific absurdities alike in the impressive uniform of formulae and theorems. Unfortunately however, an absurdity is uniform is far more persuasive than an absurdity unclad. The very fact that a theory appears in mathematical form, for instance, a theory has provided the occasion for the application of a fixed-point theorem, or of a result about difference equations, somehow makes us more ready to take it seriously. The mathematical-intellectual effort of applying the theorem fixes in us the particular point of view of the theory, making us blind to whatever appears neither as a dependent nor as an independent parameter in its mathematical formulation. The result, perhaps most common in the social sciences, is bad theory with a mathematical passport. The present point is that the application of the mathematical technique to the social sciences is bad theory with a mathematical passport. The reader prepares most commonly in the social sciences is bad theory with a mathematical passport. The reader prepares most commonly in the social sciences is bad theory with a mathematical passport.

In so large and public a gathering, however, prudence dictates the avoidance of any possible faux pas. I confine myself, therefore, to the citation of a delightful passage from Keynes' General Theory, in which the issues before us are discussed with a characteristic wisdom and wit:

"It is the great fault of symbolic pseudomathematical methods of formalizing a system of economic analysis that they express..."
The intellectual attractiveness of a mathematical argument, as well as the considerable mental labor involved in following it, makes mathematics a powerful tool of intellectual pretension—a glittering deception in which some are entrapped, and some, alas, ensnare.

Thus, for instance, the seductive ingenuity of the Birkhoff ergodic theorem has created the general impression that it must play a central role in the foundations of statistical mechanics. Let us examine this case carefully, and see what it really means.

Mechanics tells us that the configuration of an isolated mechanical system is specified by choice of a point in its phase surface, and that after a time $t$ the system moves into the configuration represented by $P$. The Birkhoff theorem tells us that if $f$ is any numerical function of the configuration $P$ (and if the mechanical system is metrically transitive), the time average of $f$ (averaged over a long period of time) is equal to the time average of $f$ (averaged over the phase surface).

The familiar argument continues, we should not expect to observe a configuration in which the long-time average of such a function $f$ is not close to its equilibrium value. Here I may conveniently use a bit of mathematical pretension of the very sort to which I object, thus paradoxically making an argument serve the purpose of its own denunciation. Let $v(e)$ denote the probability of observing a configuration in the set $e$; the application of the Birkhoff theorem just made is then justified only if $v(e) = 0$ implies $\mu(e) = 0$. The hypothesis that the configurations of a mechanical system are not concentrated on a set of small measure is necessary to make the Birkhoff theorem relevant in any sense.

This dictum is promulgated, with a characteristically straight face, in Dunford-Schwartz, *Linear Operators*, Vol. I, Chap. 7.
PHYSICAL SCIENCES

Birkhoff theorem in fact does us the service of establishing its own inability to be more than a questionably relevant superstructure upon this hypothesis. The phenomenon to be observed here is that of an involved mathematical argument hiding the fact that we understand only poorly what it is based on. This shows, in sophisticated form, the manner in which mathematics makes us blind to its own omissions - what I have elsewhere called the single-mindedness of mathematics. Typically, mathematics tells us better what to do than why to do it. Probability theory is a famous example of an example which is perhaps of the greater significance.

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